SAT MATH SECTION

Systems of Equations

System of Equations

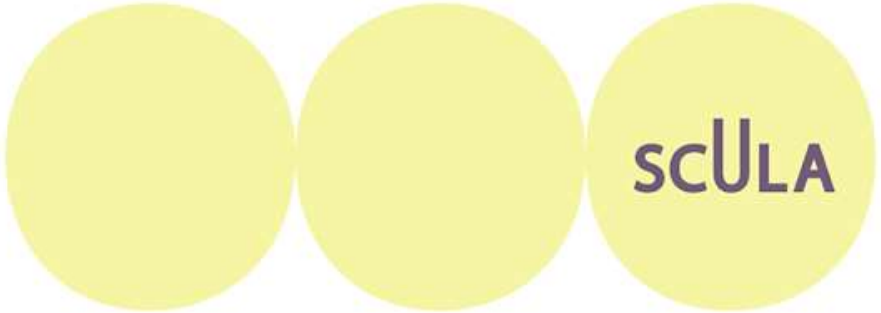
A system of equations refer to a set of two equations that deal with the same variable .

Example :

$$3x-y = 1$$

$$-x+3y= 0$$

How do we find the bundles (x,y) that verify both equations?



Substitution Method

Because the system of equations deal with two variables, x and y, we will try to adapt our system to a one single equation with only one variable .

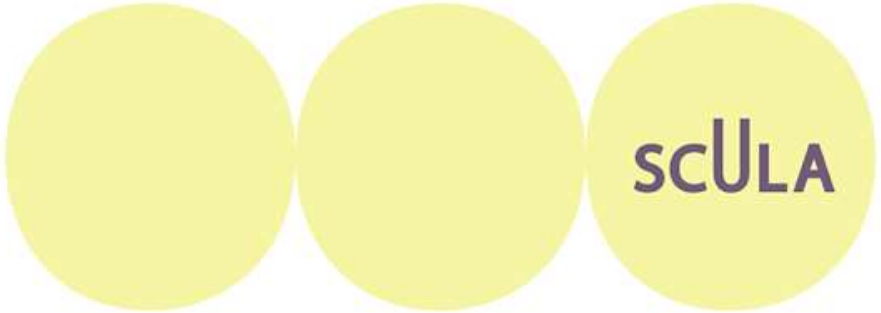
- First way to do so is to substitute one of the variables .

Example :

3 x-y = ⇒1 y = 3x - .1

We will replace y in the second equation with its expression in terms of x .

-x+3(3x -1)= 0



The given equation is a simple equation with one variable x that we can solve in the following way:

$-x+3(3x-1)=0$

$\Rightarrow -x+9x-3=0$

$\Rightarrow 8x=3$

$\Rightarrow x=\frac{3}{8}$

Then we can replace x with its value in any of the two equations and calculate y.

$3y=\frac{3}{8}$

$y=\frac{1}{8}$

The solution of the system is $(\frac{3}{8}, \frac{1}{8})$.

Elimination Method .

Another way to solve a system of equations is through elimination .

Let’s apply to the elimination method to the previous example .

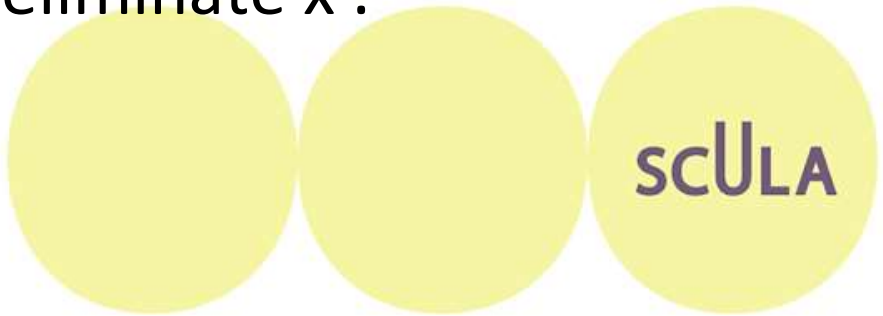
$3x-y = 1$

$-x+3y= 0$

We will multiply the second equation by 3 to eliminate x .

$3x-y = 1$

$-3x+9y= 0$



Then we will add the two equations to eliminate the variable x.

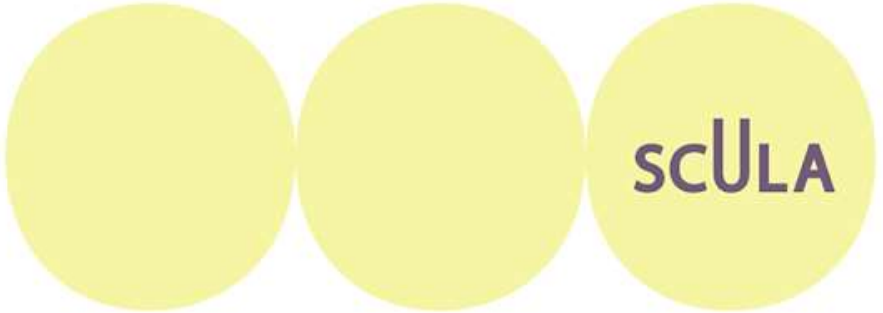
$3x - y - 3x + 9y = 1$

$8y = 1$

$y = \frac{1}{8}$

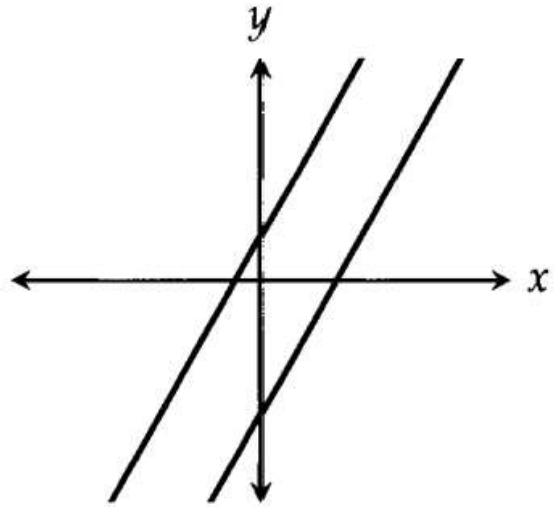
$x = \frac{3}{8}$

The solution of the system is $(\frac{3}{8}, \frac{1}{8})$.



No solution

A system of equations has no solution when the same equation is set to two different constants .



3x + 2y = 5

3x + 2y = - 4

There is no (x,y) bundle that verifies both equations.
Otherwise, this will mean .4- =5

The two lines have the same slope .

They are parallel. They never intersect .

SCULA

Let’s consider the following system of equations :

$ax + by = m$

$cx + dy = n$

If $ad - bc = 0$, then the system of equations does NOT have a solution.

$$-ax - 12y = 15$$
$$4x + 3y = -2$$

If the system of equations above has no solution, what is the value of a?

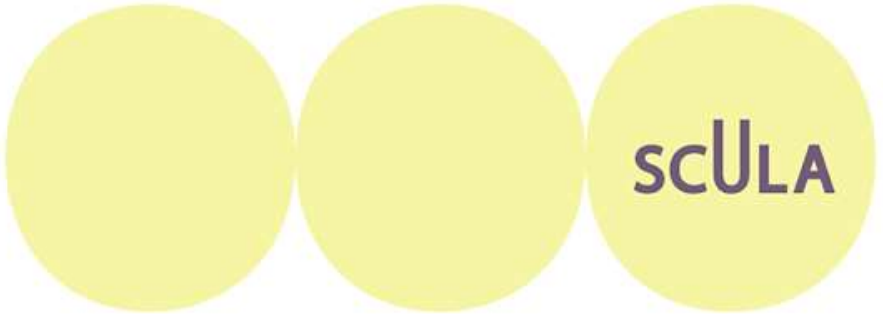
Let’s consider that the given system of equations has no solution:

-3a - (-12) ×4 = 0

⇒ -3a + 48 = 0

⇒ -3a = -48

⇒ **a = 16**



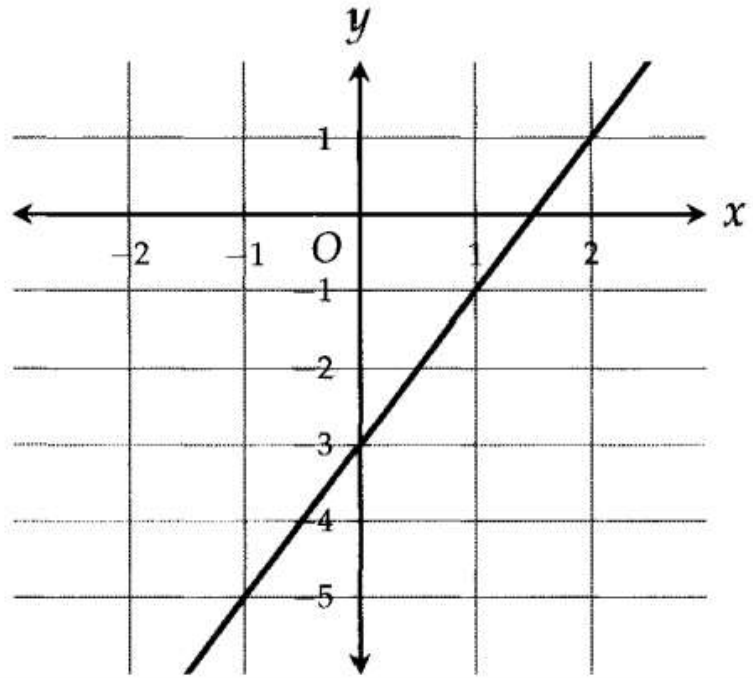
Infinite Solutions

A system has infinite solutions when the two equations are essentially the same .

Example:

$3x + 2y = 5$

$6x + 4y = 10$



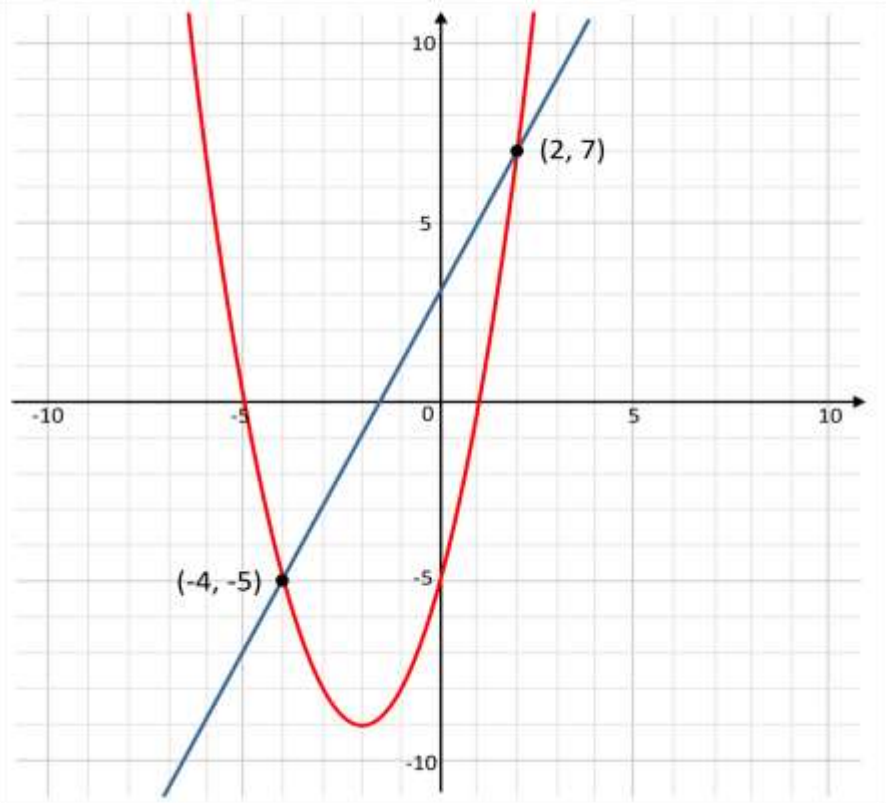
The two equations represent the same line

Therefore, there is an infinite number of real solutions.

SCULA

More Complex Systems

Some SAT questions suggest systems of equations that is not linear .
The system contains a linear equation and a quadratic equation .



In this case, substitution is the most efficient way to solve this type of problem .

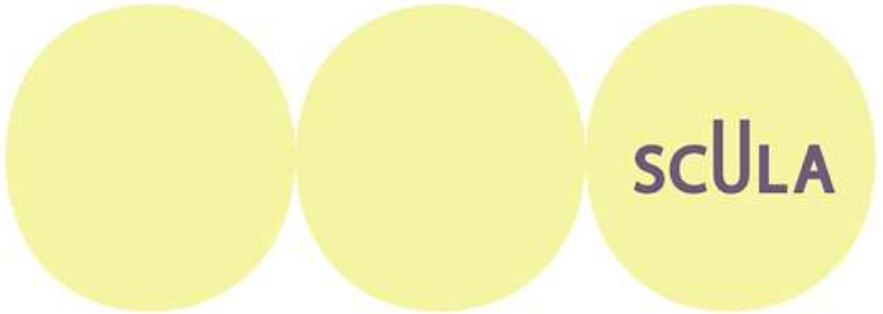
SCULA

$$y + 3x = 0$$
$$x^2 + 2y^2 = 76$$

If (x, y) is a solution to the system of equations above and $y > 0$, what is the value of y?

We isolate y in the first equation and replace it in the second equation y = -3x

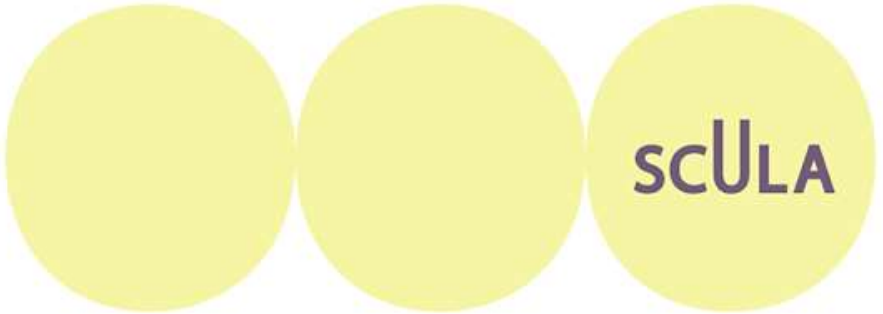
$$x^2 + 2(-3x)^2 = 76$$
$$x^2 + 2(9x^2) = 76$$
$$x^2 + 18x^2 = 76$$
$$19x^2 = 76$$
$$x^2 = 4$$
$$x = \pm 2$$



If $x = 2$, then $y = -3(2) = -6$. If $x = -2$, then $y = -3(-2) = 6$. Because $y > 0$, $y = \boxed{6}$.

For this type of systems, substitution and some equation manipulation should do the trick. If the question asks for one possible value, any of the two solutions will be considered correct .

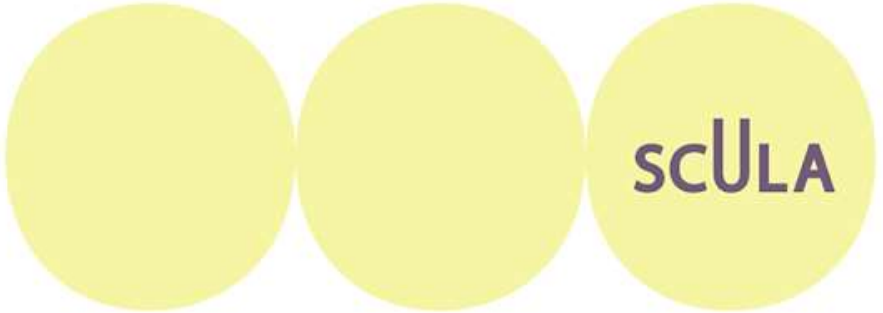
If the question asks for the positive value, as in the example, then only the positive value will be considered a correct answer ,



Word Problems

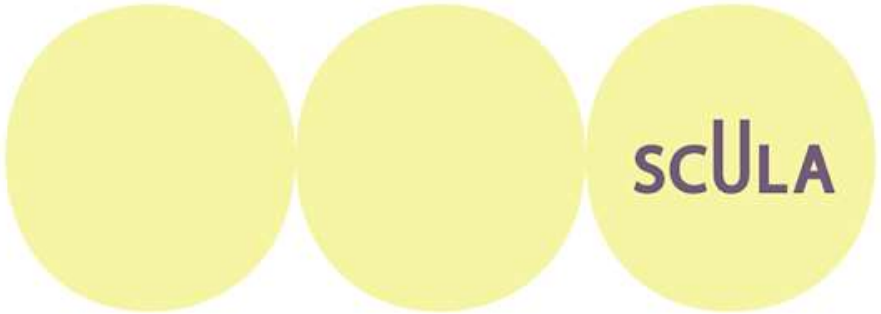
Some word problems require you to translate the given information into a system of equations in order to solve the suggested question

Translating a word problem into an equation is the most important skill you need to develop to nail the SAT Math Section .



Blin and Alex raised a total of $240,000 for a charity. If Blin raised $60,000 more than Alex did, how much money did Blin raise?

This problem provides you with a classic SAT Systems of Equations setup. You're given the sum of two entities - here that's **Blin's Total + Alex's Total = $240,000.** And a relationship between the two totals; **Blin raised $60,000 more than Alex did.**



Now you have two equations:

B + A = 240,000

B = 60,000 + A

This structure sets up well for Substitution; you already have B in terms of A, so you can plug in the second equation for the first:

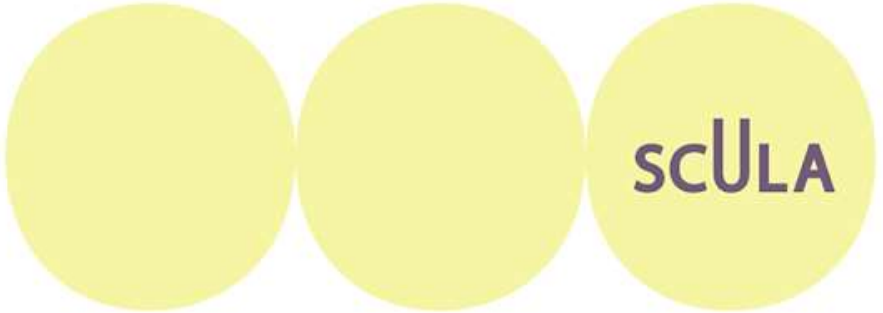
(60,000 + A) + A = 240,000

60,000 + 2A = 240,000

2A = 180,000

A = 90,000

Remember the question asked how much money did Blin raise?



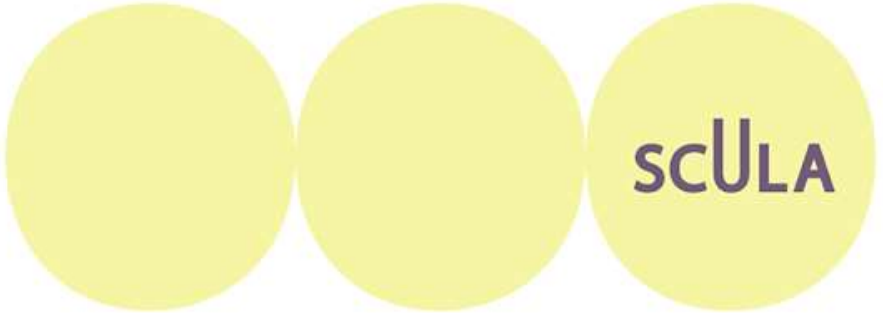
One of the tricks of the SAT is to have 90,000 as one of the answer choices. If you are stressed and in a hurry, you are most likely to pick that answer before you answer the real question.

The first equations gives us the relationship between A and B:

B + A = 240,000

B = 240,000 - A

B = 15,000



Another Example..

Siblings Tamika and David tracked every hour they spent studying during a particular school year. Combined, they studied for a total of 932 hours, with Tamika studying for 28 more hours than her brother David did. How many hours did Tamika study?

Let’s set the equations :

$T + D = 932$

Then the second equation is:

$T = D + 28$. (you can think of this as "to equal Tamika's total, David would have had to add 28 hours")

Now you can stack your equations and sum them:

$T + D = 932$

$T - D = 28$

These sum to:

$2T = 960 \rightarrow$ **$T = 480$**

PRACTICE

https://drive.google.com/file/d/12tCEXfQPgC6XYahD6XRUQF51zOO-6dTp/view?usp=drive_link

THANK YOU!

DO YOU HAVE ANY QUESTIONS?