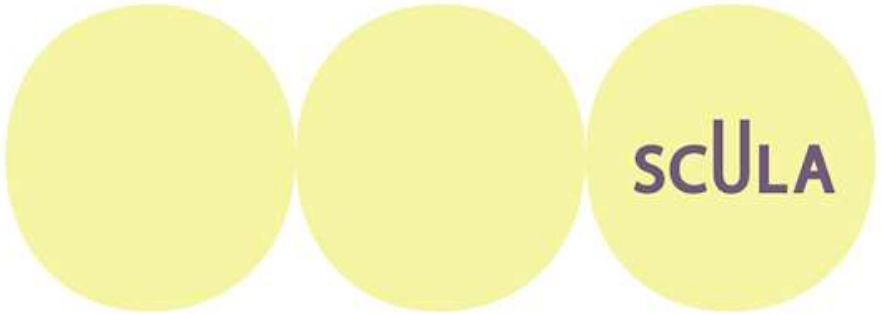
SAT MATH SECTION

Rates, Proportions, and Percent

What is the difference between a rate and a percent?



Let’s set the basics..

A ratio is a comparison of two quantities by division. I.e 3:5

A rate is a specific type of ratio that compares two quantities with different units. I.e 3km/h.

You drive 60 miles in 2 hours, the rate of your speed is 60 miles per 2 hours, or 30miles per hour.

A proportion is an equation that states that two ratios are equal .

$$\frac{60\ miles}{2\ hours} = \frac{30\ miles}{1\ hour} = 30 miles/hour$$

SCULA

Percent Change

We use the percent (or percentage) to estimate the change happening in different values with respect to new events .

Let's discuss the notion of percent

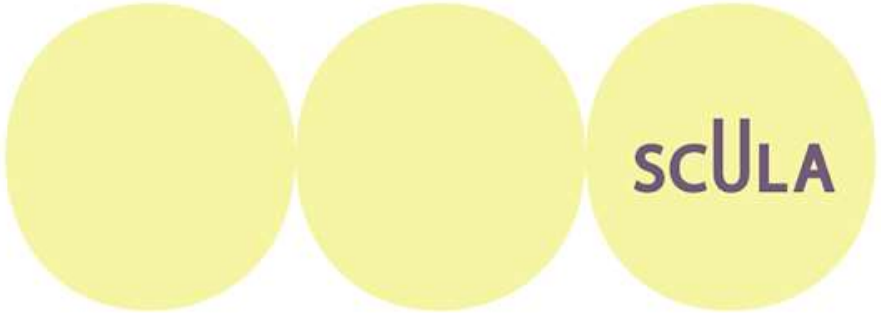
EXAMPLE 1: Jacob got 50% of the questions correct on a 30-question test and 90% on a 50 question test. What percent of all questions did Jacob get correct?

- **For each 100 questions : 50 were correct.**

How many were correct in a ?tset notiseuq-30

- **For each 100 questions : 90 were correct .**

How many were correct in a ?tset notiseuq-50



First, let's find the total number of questions he got correct:

$$50\% \times 30 = \frac{1}{2} \times 30 = 15$$

$$90\% \times 50 = \frac{9}{10} \times 50 = 45$$

So he got $15 + 45 = 60$ questions correct out of a total of $30 + 50 = 80$ questions. $\frac{60}{80} = \frac{3}{4} = \boxed{75}\%$

On the first test :

50 → 100

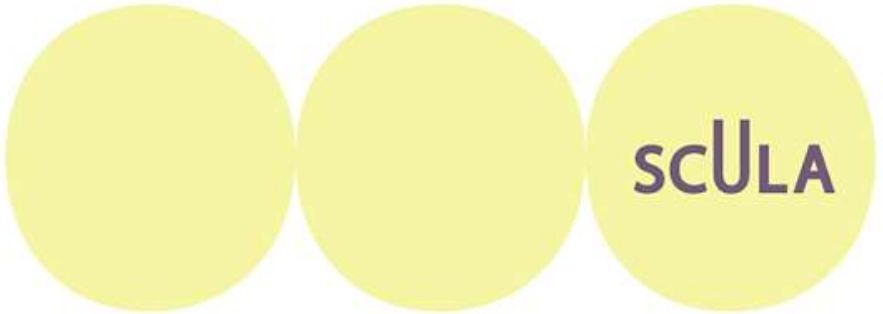
? → 30

For the second test :

90 → 100

? → 30

This value is what we call The Value of Change



Percent Change:

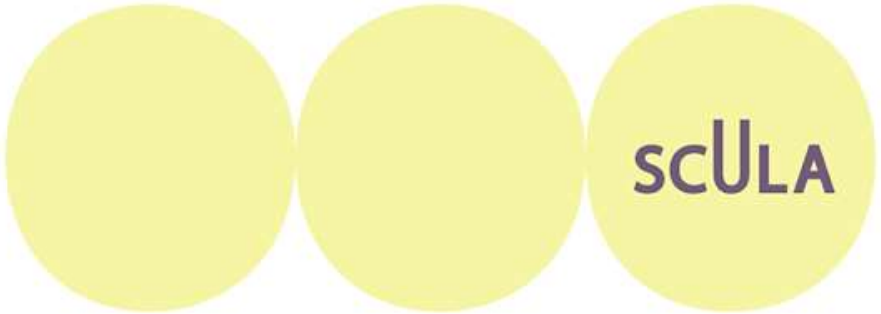
Original Value (A) → **New Value (P)**

The change can either be an increase or a decrease annotated as r %

If the change is an increase (or a positive change), we will add the value of change .

$$A = P + P \times \frac{p}{100}$$

$$A = P\ (1 + \frac{p}{100})$$



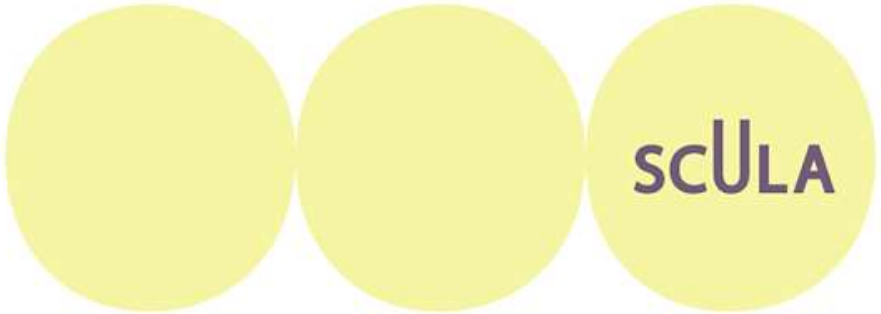
If there is a decrease in the original value, the new value is expressed as follow :

$$A = P\left(1 - \frac{r}{100}\right)$$

From the two expressions above, we can deduce the expression of r .

$$r = \frac{A - P}{P} \times 100$$

This expression is the percent change .



Note: The formula for the simple increase or decrease does not still applies in the case of consecutive changes to the same original value .

EXAMPLE 3: The price of a dress is increased by 20%, then decreased by 40%, then increased by 25%. The final price is what percent of the original price?

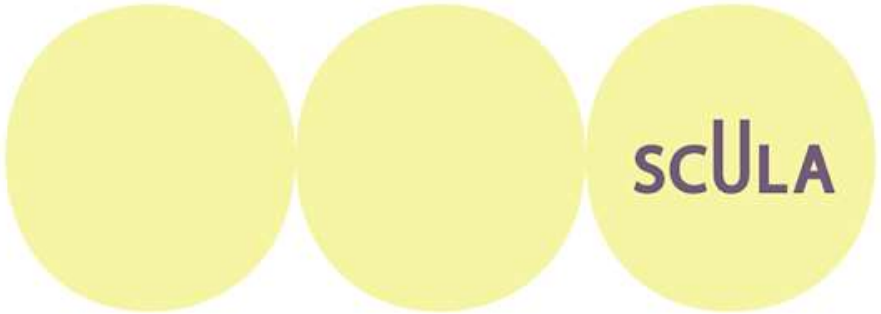
$A(0.2 + 1)A =_1$

$A(0.4 - 1)_1A =_2$

$A(0.25 + 1)_2A =_3$

$A(0.25 + 1)\ (0.4 - 1\ )_1A =_3$

$A(0.25 + 1)\ (0.4 - 1\ )\ (0.2 + 1\ )A =_3$



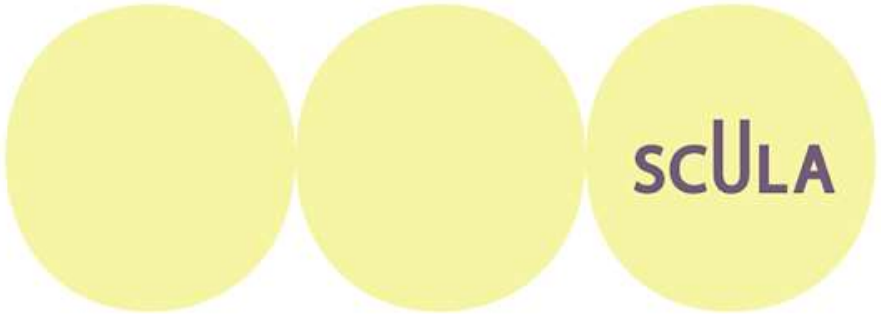
The simple Interest

An investor decides to offer a business owner a**20 000 $** loan at a simple interest rate of**5% per year.** What is the total amount the investor will receive after t years.

Total Amount = Original Value + Interest Value (for t years)

Total Amount = 20 000 + 0.05t × 20 000

Total Amount = 20 000 (1 + 0.05t)

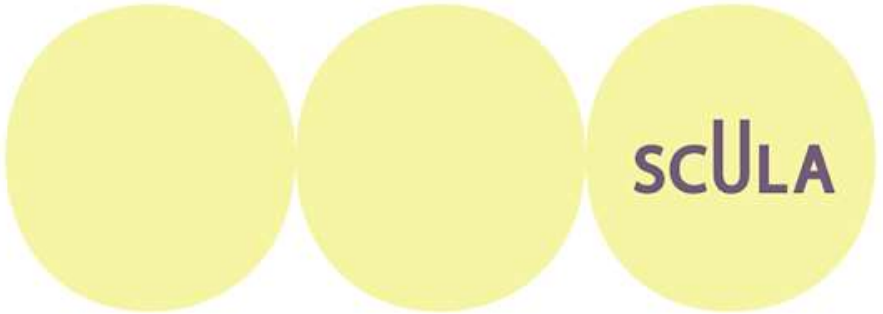


The simple Interest Formula

The simple interest : an interest rate r on the original value for t years .

Total Amount = Original Value + Interest Value

Total Amount = A(1 + rt)



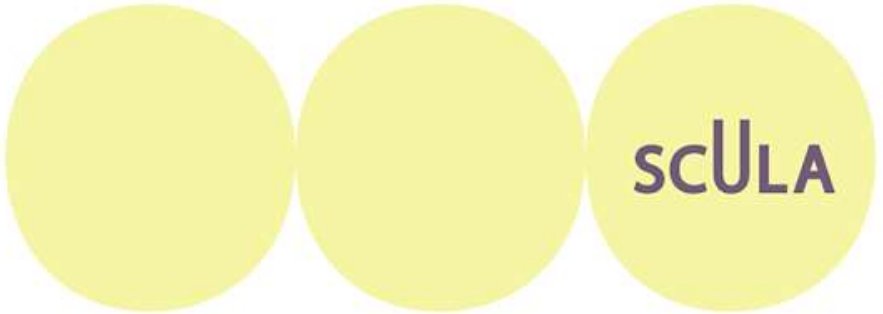
The compound Interest

John has a saving account where deposited $1000 with a compound interest rate of 5% annually. What is the value of his account after 3 years ?

Year 1: 1000(1.05)

Year 2 : 1000(1.05) + 0.5[1000(1.05)] = 1000(1.05)(1.05)

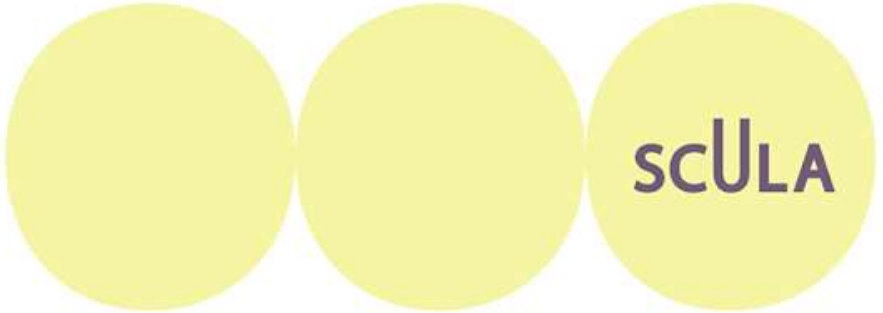
Year 3 : 1000(1.05)(1.05)(1.05)



The compound Interest Formula

Compound interest: **an interest rate r on the previous year's value for t years.**

$$Total\ Value = A(1 + r)^t$$



Interest Compounded n times over a period t .

If the interest is compounded more than once in a year (or any unit period t), the previous formula will be generalized to :

$$Total\ Value = A(1 + \frac{r}{n})^{nt}$$

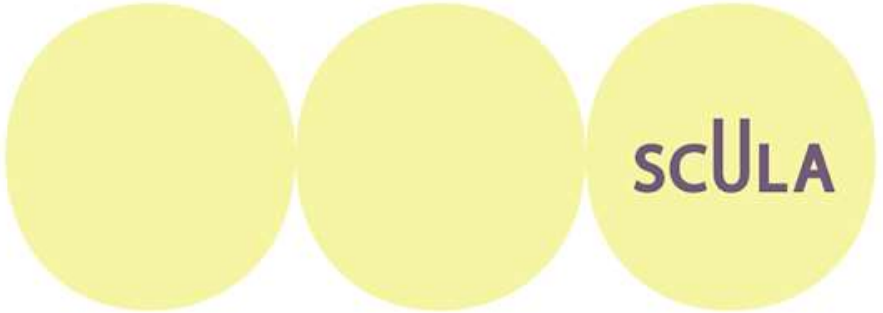
EXAMPLE 5: Jay puts an initial deposit of \$400 into a bank account that earns 5 percent interest each year, compounded semiannually. Which of the following equations gives the total dollar amount, A, in the account after t years?

A) $A = 400(1 + 0.05t)$ B) $A = 400(1 + 0.1t)$ C) $A = 400(1.05)^t$ D) $A = 400(1.025)^{2t}$

The interest is compounded semiannually, which means twice in one year.

$$Total\ Value = 400(1 + \frac{0.05}{2})^{2t}$$

$$Total\ Value = 400(1.025)^{2t}$$



Recap…

Value of change

Percent change

String of Changes

Simple Interest

Compound Interest

Interest Compounded n times over a period t .

PRACTICE

https://drive.google.com/file/d/1R6gzp3j_jeE-39uLteGEhZKk_9_vKOIw/view?usp=drive_link

THANK YOU!

DO YOU HAVE ANY QUESTIONS?