SCULA

مركز الإرشاد العالمي للتعليم

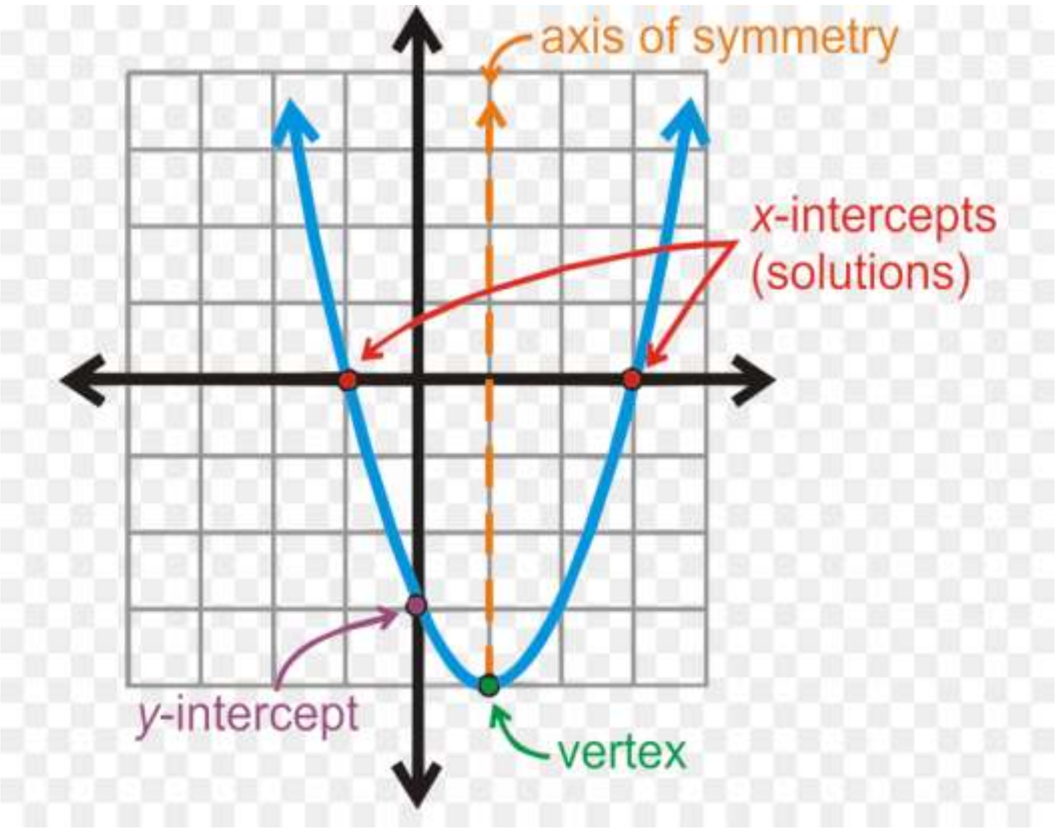
SAT MATH SECTION

Quadratic Functions

What is a Quadratic ?

$$Ax^2 + Bx + C= 0$$

Quadratic equation is any equation written in the form below, with A, B, and C are real numbers .



To manipulate a quadratic equation, we will assess the following characteristics

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ROOTS = x-intercepts = solutions

(E) : $x^2 - 4x - 21 = 0$

$\Leftrightarrow (x+3)(x-7) = 0$

$\Leftrightarrow x+3 = 0$ or $x-7 = 0$

$\Leftrightarrow x = -3$ or $x = 7$

-3 and 7 are called the roots of the equation (E)

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THE SUM AND THE PRODUCT OF THE ROOTS

$$Ax^2 + Bx + C = 0$$

The sum :

$$x_1 + x_2 = -\frac{B}{A}$$

The product :

$$x_1 \times x_2 = \frac{C}{A}$$

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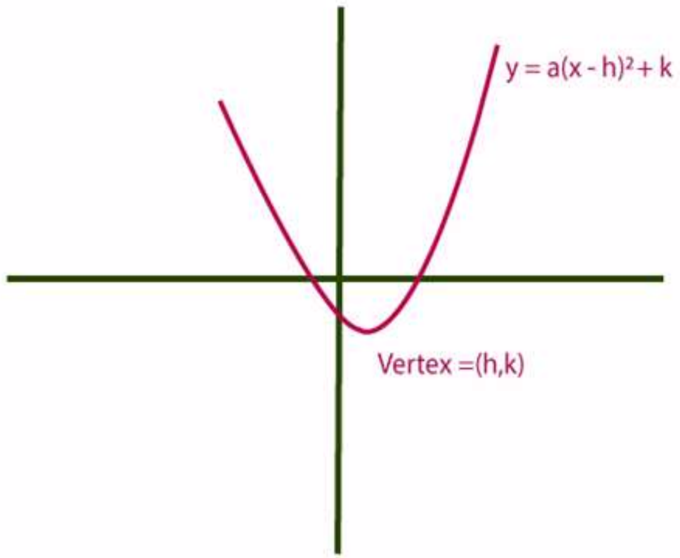
tniop xetrev eht si (k,h)

The point (h,k) is the vertex of the quadratic function .

It represents either the min or the max value of the function .

To find the vertex algebraically, we need to write the function in the vertex form.



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The standard form of the quadratic equation : $f(x) = ax^2 + bx + c$

The vertex form of a quadratic equation : $f(x) = a(x - h)^2 + k$

Example

$$y = x^2 - 4x - 21$$

$$y = x^2 - 4x + 4 - 4 - 21$$

$$y = (x - 2)^2 - 25$$

$$The\ vertex\ point\ is\ (2,\ -25)$$

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The axis of symmetry of a quadratic

The quadratic function, as shown in the graph, is an even function. Let's consider x and x 'the roots of a quadratic function .

$$f(x) = ax^2 + bx + c$$

$$h = \frac{x + x'}{2}$$

$$k = f(h)$$

The axis of symmetry passed through the vertex point.

Later, we will encounter a similar formula of the x-coordinate of the vertex point .

Discriminant

The value of the discriminant of a quadratic equation is written as follow:

$$D = b^2 - 4ac$$

The sign of the discriminant indicates the number of solutions of the equation .

D > :mrof siht ni snoitulos eht etirw nac eW .snoitulos owt sah noitauqe ehT :0

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

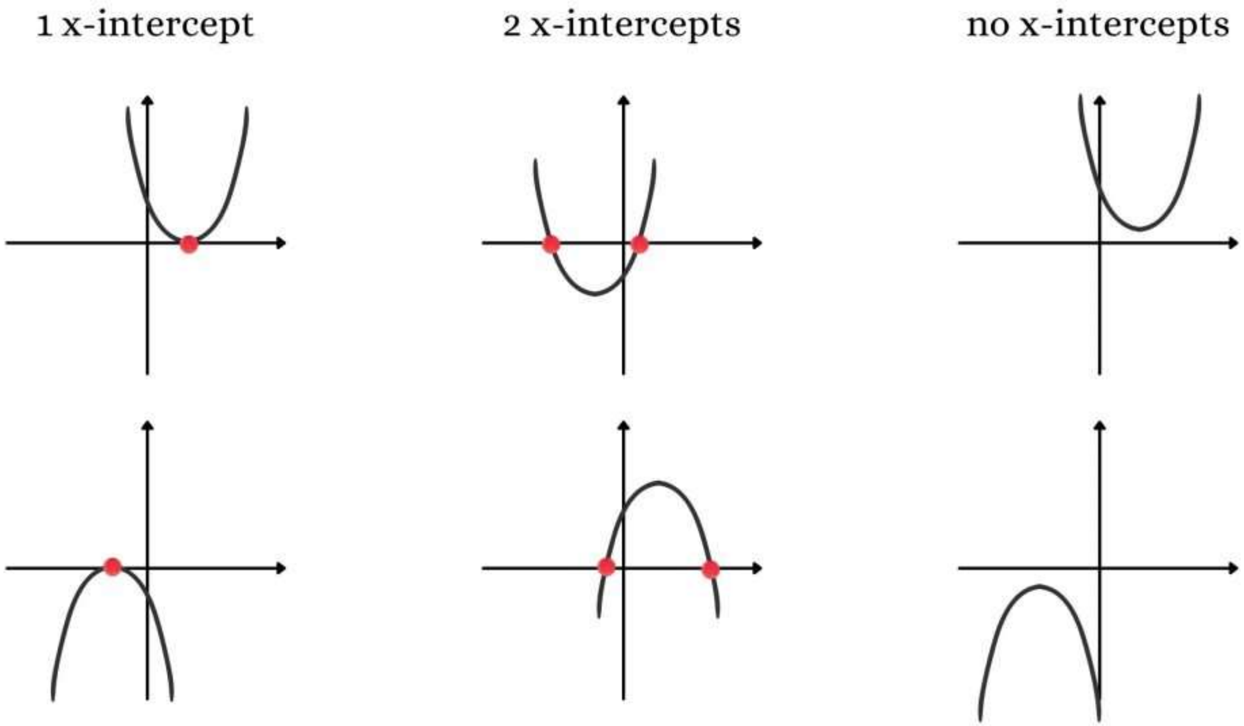
D < .noitulos laer a evah ton seod noitauqe ehT :0

D = .noitulos laer eno ylno sah noitauqe ehT :0

$$x = -\frac{b}{2a}$$

Graphically

As we discussed before, the solution (x-intercept) represents the intersection of the graph with the x-axis .



In certain cases, it is easier to factor the equation instead of using the discriminant. But keep in mind that if you prefer one method, the results will remain the same.

$$64x^2 + 24x - 10 = 0$$

$$(8x + 5)(8x - 2) = 0$$

$$8x + 5 = 0 \quad or \quad 8x - 2 = 0$$

$$x = -\frac{5}{8} \quad or \quad x = \frac{2}{8} = \frac{1}{4}$$

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More Examples

EXAMPLE 1: In the xy-plane, the parabola with equation $y = x^2 - 5x + 6$ intersects the line $y = 3x - 10$ at point (a, b). What is the value of b?

$$\begin{cases} y = x^2 - 5x + 6 \\ y = 3x - 10 \end{cases}$$

The two equations intersect at one point (a, b)

$$x^2 - 5x + 6 = 3x - 10$$

$$x^2 - 8x + 16 = 0$$

$$(x - 4)^2 = 0$$

$$x - 4 = 0$$

$$x = 4 = a$$

$$b = 12 - 10 = 2$$

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EXAMPLE 4: A biologist uses the function $p(n) = -100n^2 + 1{,}000n$ to model the population of seagulls on a beach in year number n, where $1 \leq n \leq 10$. Which of the following equivalent forms of $p(n)$ displays the maximum population of seagulls and the number of the year in which the population reaches that maximum as constants or coefficients?

(A) $p(n) = -4n(25n - 250)$ (B) $p(n) = -10(10n^2 - 100n)$ (C) $p(n) = -100(x - 5)^2 + 2{,}500$

(D) $p(n) = -100(x - 7)^2 + 4{,}900$

Anytime you see the quadratics questions asking for the maximum or the minimum, try to find the vertex coordinates or the vertex form.

The question requires the maximum population while p(n) represents the population in n years .

$$-100n^2 + 1000n = 0$$

$$-n^2 + 10n = 0$$

$$n(10 - n) = 0$$

$$n = 0 \ or \ n = 10$$

The population reaches 0 at n = 0 or n = 10.

$$h = \frac{10}{2} = 5$$

$$p(5) = 2500$$

Thus, the maximum polupation is 2500.

You can also reach this result by writing the equation in the vertex form.

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IMPORTANT NOTE

We know now the expression of the two solutions of a quadratic equation :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We also know that the x-coordinate of the vertex is the middle of x-intercepts.

$$h = \frac{x_1 + x_2'}{2}$$

$$h = \frac{1}{2}\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} + \frac{-b + \sqrt{b^2 - 4ac}}{2a}\right) = -\frac{b}{2a}$$

$$k = f\left(-\frac{b}{2a}\right)$$

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PRACTICE

https://drive.google.com/file/d/1y9tDGyRBPdxywjhQ4tkZcD3zaaAvR2tn/view?usp=drive_link

THANK YOU!

DO YOU HAVE ANY QUESTIONS?