SAT MATH SECTION

Circles

After Angles and Triangles, we will now go over Circles .

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Today, we will focus on problem sets as we discuss the module content .

Some of this information will be helpful to understand trigonometry. Make sure you pay close attention .

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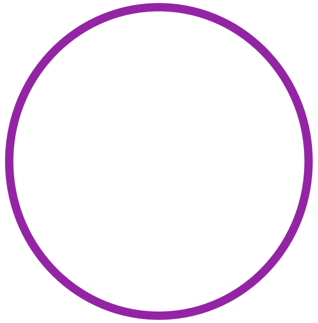
Circles

- Area and Circumference
- Arc Length and Arc Measure
- Area of a Sector
- Inscribed Angles
- Equations of Circles

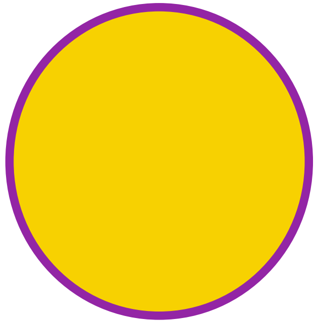
What is a circle ?

What is the measure of the angle d?



This is a circle.

When we talk about a circle, we mean the circumference in purple, the length that is surrounding the inside area.



This is a disk.

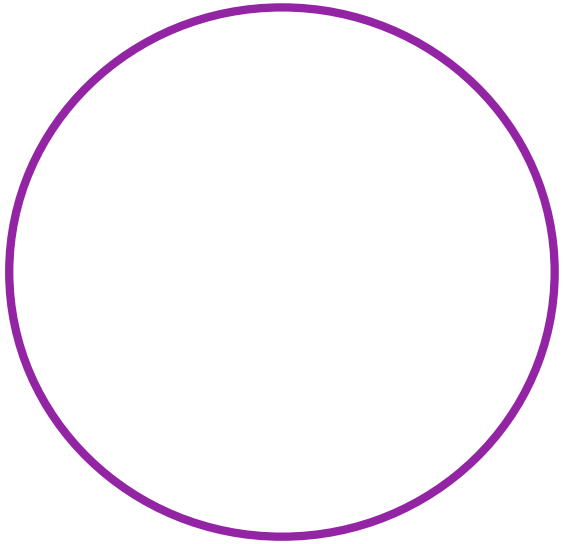
A disk refers to the inside area

of a circle .

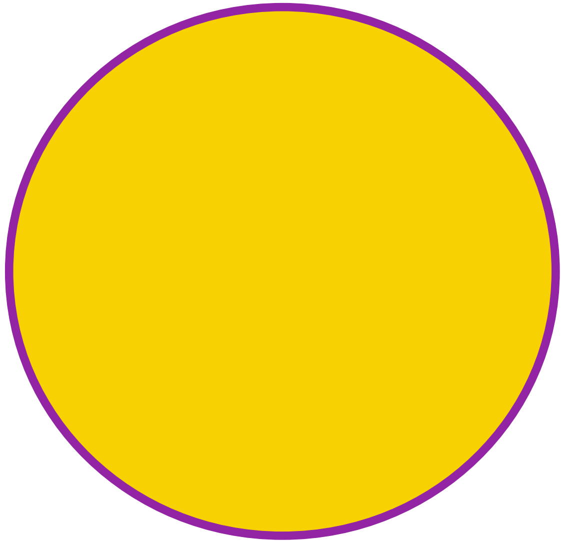
The circumference :

$A = \pi R^2$

Where R is the radius of the Circle.



The area :



$$A = 2\pi R$$

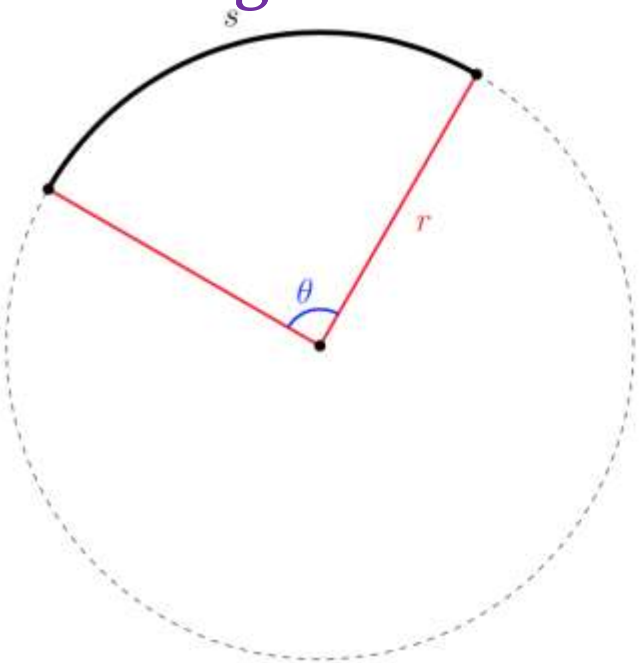
Where R is the radius of the Circle.

Arc Length :

The arc length is the length (distance) of the arc AB where A and B are points from a circle.

To measure this length, we will discuss the concept of radian.

We know that π equates 180 degrees. Let θ be the angle inside the arc.



$2\pi R \rightarrow 180 \times R$
$2\pi R \rightarrow \theta \times R$

ARC LENGTH = $\theta \times R$

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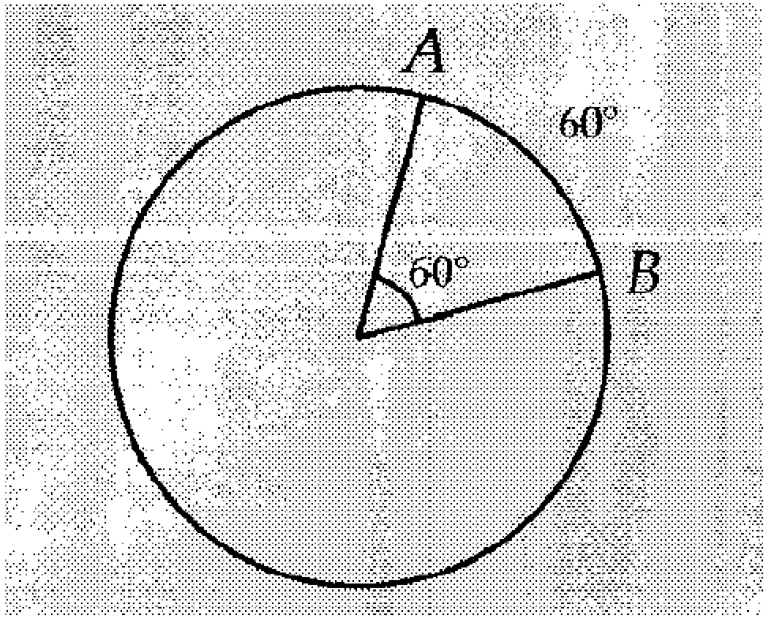
Arc measure :

The arc measure is NOT the same as the arc length.

The arc measure signifies the measure of the angle inscribed in the arc.

The arc measure represents **θ** usually in radians.



ARC LENGTH = ARC MEASURE × R

We call these angles **central angles**.
They have the same measure as the arcs that they carve out

The Area of a Sector :

$2\pi \rightarrow \pi R^2$

$\theta \rightarrow A$

Using the proportionality relationship:

$$A = \frac{1}{2} R\theta^2$$

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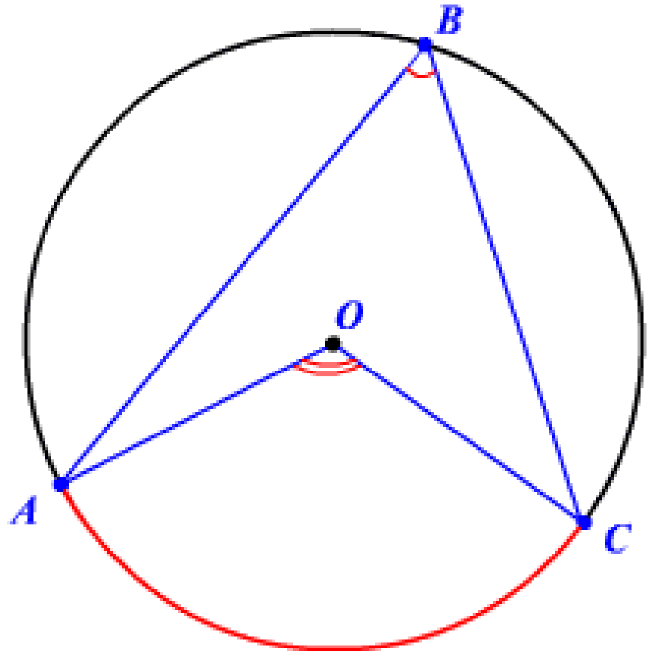
Inscribed Angle :

An inscribed angle in a circle is formed by two chords that have a common endpoint on the circle. This common end point is the vertex of the angle.

<u>The measure of an inscribed angle is half the measure of the intercepted arc.</u>

$\angle ABC = \frac{1}{2} \angle AOC$

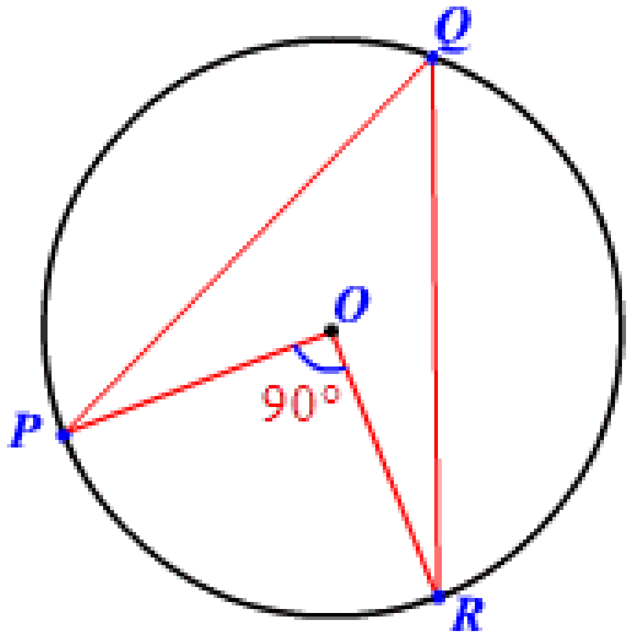


Find the measure of the inscribed angle $\angle PQR$.

By the inscribed angle theorem, the measure of an inscribed angle is half the measure of the intercepted arc.

The measure of the central angle $\angle PQR$ of the intercepted arc

PR is $90°$



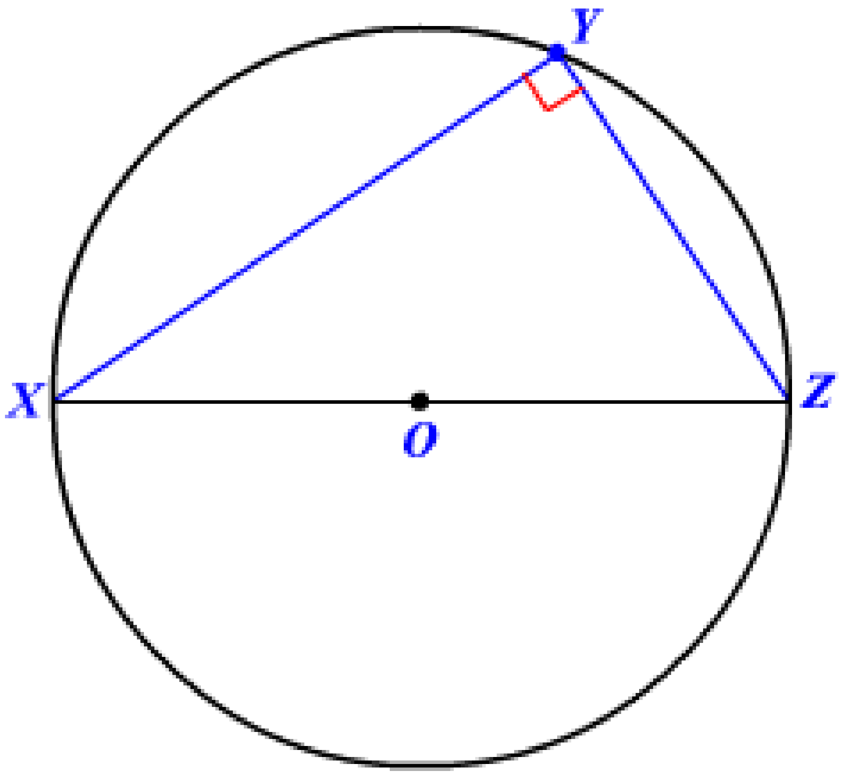
Therefore,

$\angle PQR = \frac{1}{2} \angle POR$

$\angle PQR = 45°$

Inscribed Angle

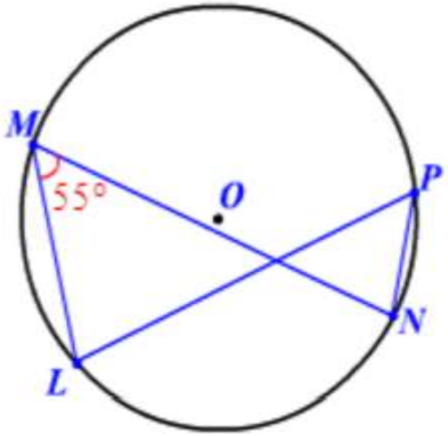
An especially interesting result of the Inscribed Angle Theorem is that an angle inscribed in a semicircle is a right angle.



In a semicircle, the intercepted arc measures 180° and therefore any corresponding inscribed angle would measure half of it

EXAMPLE

Find $m\angle LPN$.



In a circle, any two inscribed angles with the same intercepted arcs are congruent.

Here, the inscribed angles $\angle LMN$ and $\angle LPN$ have the same intercepted arc $\overparen{LN}$.

So, $\angle LMN \cong \angle LPN$.

Therefore, $m\angle LMN = m\angle LPN = 55^\circ$.

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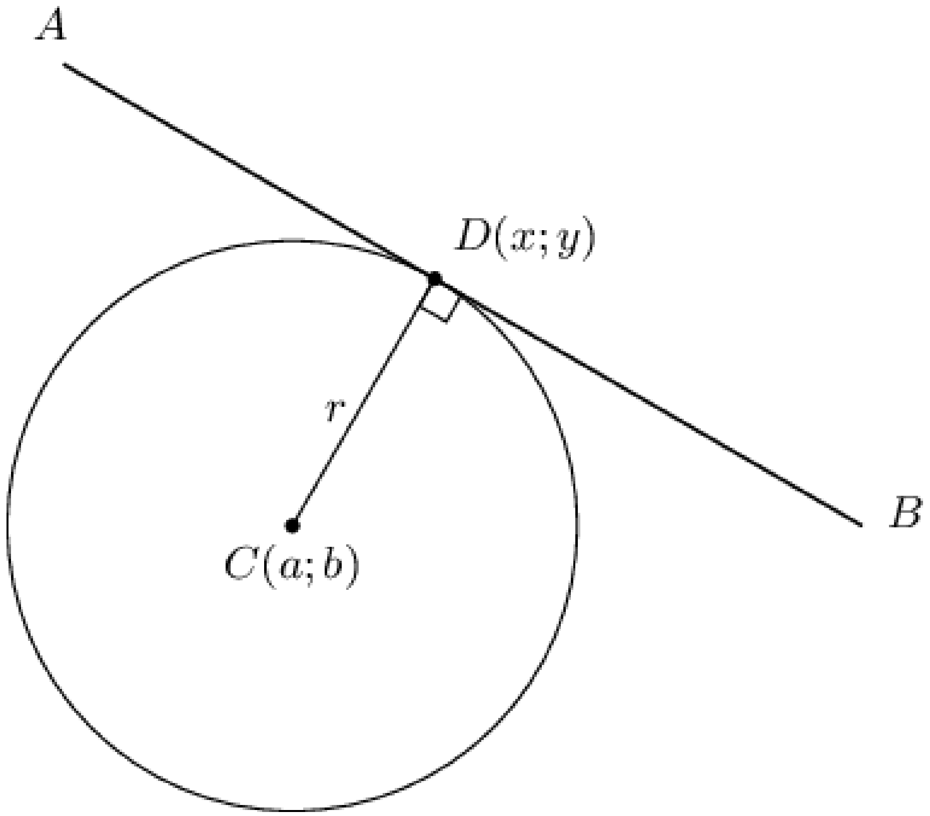
Tangent Line



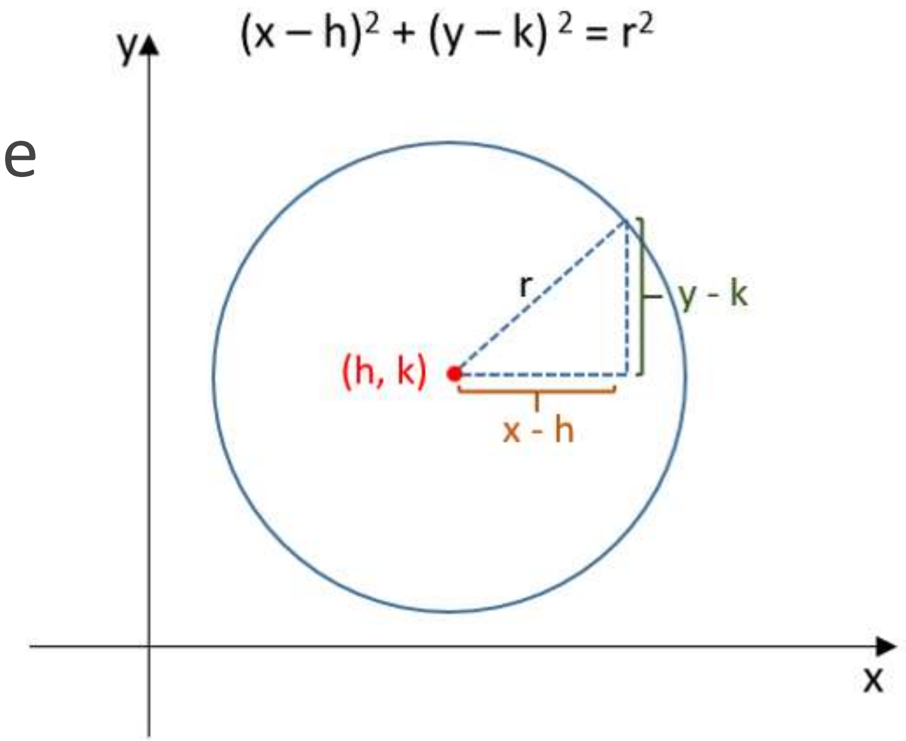
A radius drawn to a line tangent to the circle is perpendicular to that line .

For instance, a deduction that can be made here is that ADC and DBC are right triangles at the vertex D

The standard equation of a circle is given by:

$(x-h)^2 + (y-k)^2 = r^2$

Where (h,k) is the coordinates of center of the circle and r is the radius.



Example

$(x-1)^2+(y-2)^2 = 4^2$

$(x^2-2x+1)+(y^2-4y+4) = 16$

$x^2+y^2-2x-4y-11 = 0$

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Example

What is the radius of a circle with the equation $x(x-8)+y(y-6)=24$?

We need to expand this equation to $x^2-8x+y^2-6y=24$ and then complete the square.

This brings us to $x^2-8x+16+y^2-6y+9=24+16+9$.

We simplify this to $(x-4)^2+(y-3)^2=49$.

Thus the radius is 7.

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Next session, we will do practice problem to apply everything we have seen so far. Make sure to review Angles, Triangles, and Circles to be able to solve problem sets .

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THANK YOU!

DO YOU HAVE ANY QUESTIONS?