

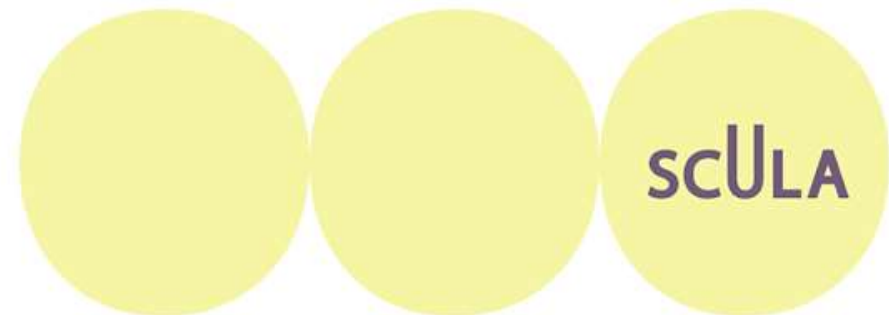
SAT MATH SECTION

Complex Numbers



Today, we will discuss a new concept .

- Definition of Complex Numbers
- Sum of Complex Numbers
- Product of Complex Numbers
- Conjugate of a Complex Number



REAL NUMBER vs . COMPLEX NUMBERS



We will start with a familiar situation.

Remember the quadratic functions that did not have any real solutions?

This is because they came down to an equation that looks like this:

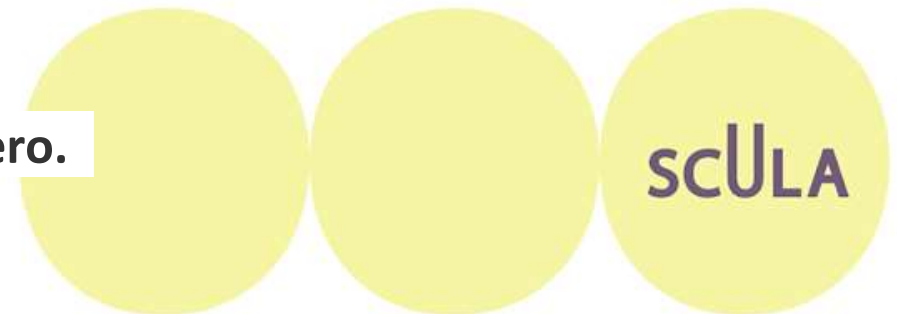
$$x^2 + 1 = 0$$

$$x^2 = -1$$

This equation, indeed, does not have real solutions.

There is no real number squared that is equal to -1 or any

negative number. Squared numbers are positive or equal to zero.



Introduction of the number *i*

To solve similar equations, Gerolamo Cardano invented an imaginary number, referred to as *i* for which this previous equation applies.

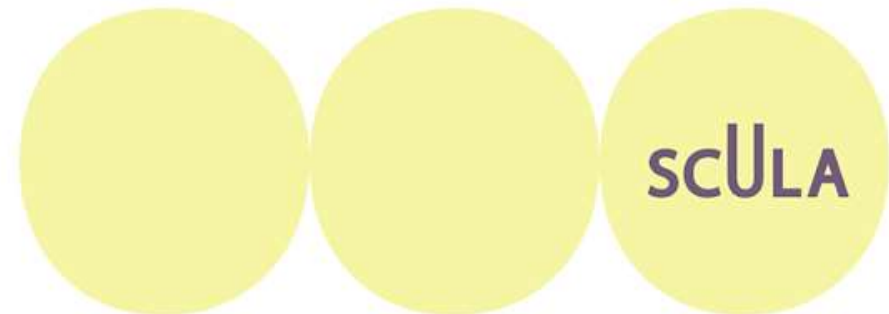
$$i^2 = -1$$

With the introduction of this number, we can find complex solutions that are made of the imaginary number *i*.

For example,

$$x^2 = -5 = -1 \times (\sqrt{5})^2$$

$$x^2 = -5 = i^2 \times (\sqrt{5})^2$$



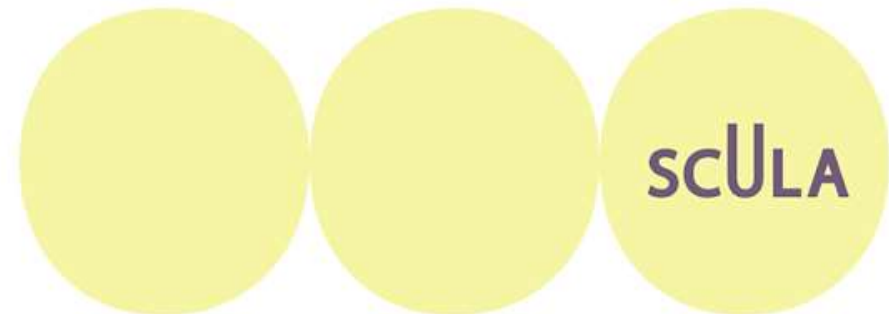
Therefore ,

$$x = i5 \sqrt{} \text{ or } x = -i5\sqrt{}$$

The previous equation has complex solutions but no real solutions .

$i5 \sqrt{}$ is a complex number .

In general, any number written in the form of $z = a + ib$ where a and b are real numbers is called a complex number



From there, any power of i can be defined as follow :

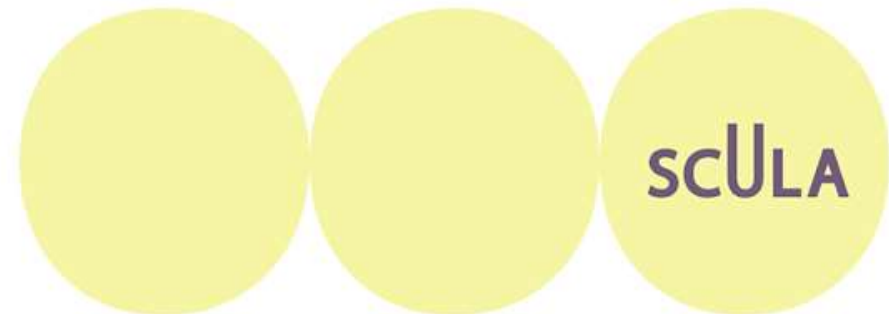
$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

$$i^5 = i$$

$$i^6 = -1$$



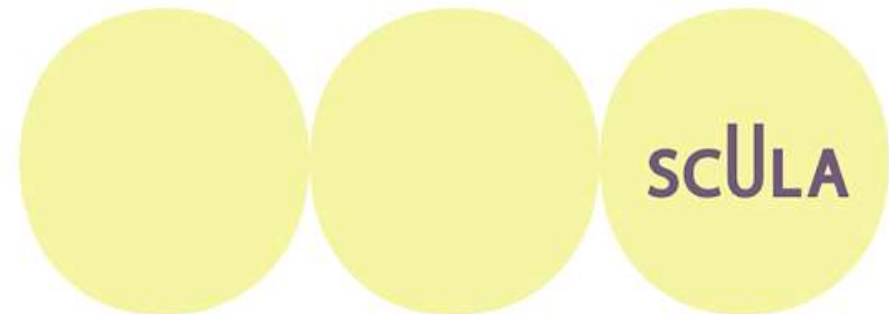
$z = a + ib$ is a complex number.

a is called the real part (because a is a real number)

ib is the imaginary part (because b is multiplied by the imaginary number i).

Every real number is a complex number with an imaginary part equal to 0.

Complex numbers cannot, on the other hand, be real numbers.



The Sum of Complex Numbers:

The sum of two complex numbers is a complex number.

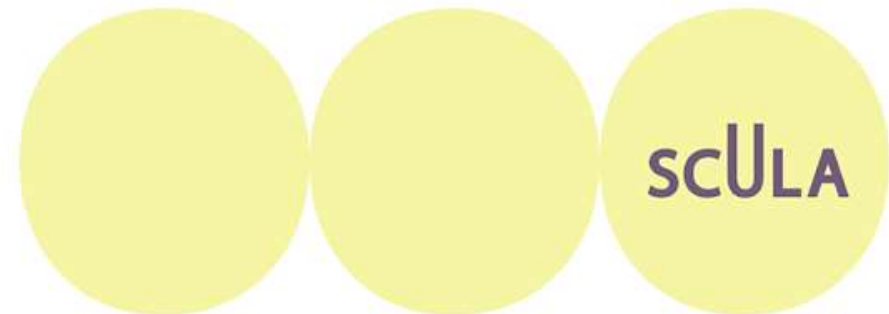
Let's consider z and z' two complex numbers.

$$z = a + ib$$

$$z' = a' + ib'$$

$$z + z' = a + ib + a' + ib'$$

$$z + z' = (a + a') + i(b + b')$$



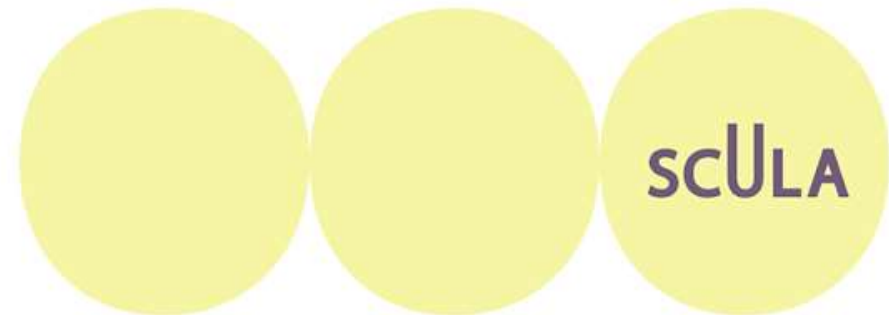
EXAMPLE 1: If $i = \sqrt{-1}$, which of the following is equivalent to $(3 + 5i) - (2 - 3i)$?

- A) $9i$ B) $1 + 2i$ C) $1 + 8i$ D) $5 + 8i$

The sum and the subtraction of two complex numbers or more follows the same rules. We sum the real parts and imaginary parts to have a complex number in the form of $a + ib$

If $a = 0$ then the complex number will be in the form of ib .

If $b = 0$ then the complex number will be a real number a



We need to expand and combine like terms as usual :

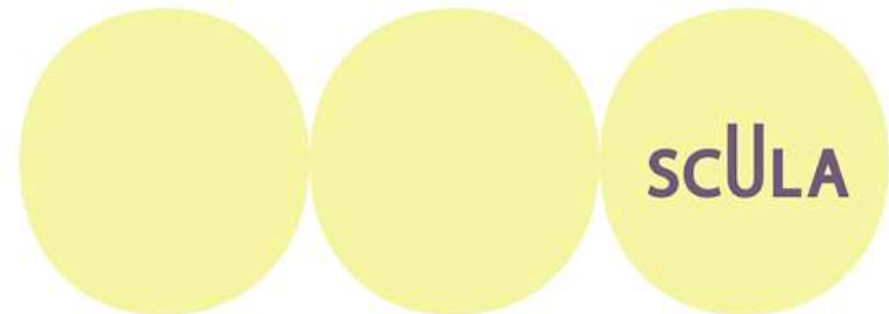
$$S = (3 + 5i) - (2 - 3i)$$

$$S = 3 + 5i - 2 + 3i$$

$$S = 1 + 8i$$

Therefore, the answer is C

Remember the SAT will only require you to manipulate the sum or subtraction of complex numbers.



The Product of Complex Numbers:

The product of two complex numbers is also a complex number.

Let's consider z and z' two complex numbers.

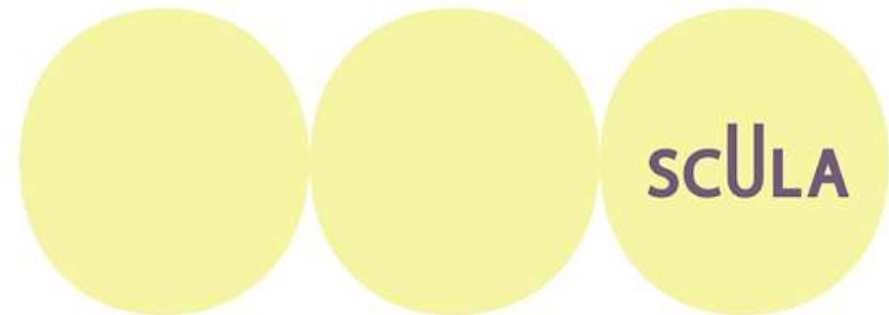
$$z = a + ib$$

$$z' = a' + ib'$$

$$z \times z' = (a + ib)(a' + ib')$$

$$z z' = aa' + iab' + ia'b - bb'$$

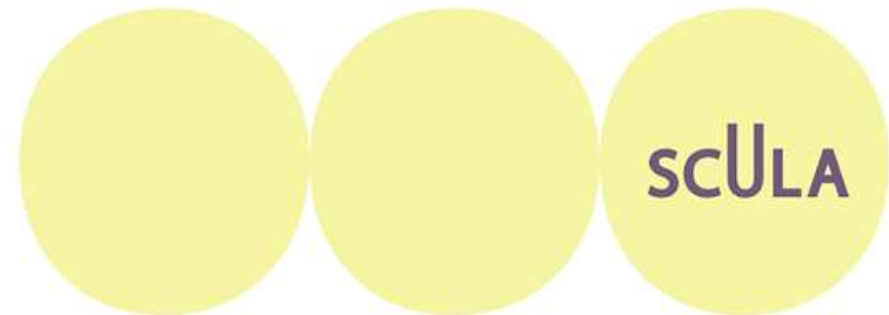
$$z z' = (aa' - bb') + i(ab' + a'b)$$



EXAMPLE 2: Given that $i = \sqrt{-1}$, what is the product $(4 + i)(5 - 2i)$?

- A) $18 - 3i$ B) $22 - 3i$ C) $18 + 3i$ D) $22 + 3i$

We deal with the product, as we discussed, in the same way that we expand any product. Then we combine like terms to have a complex number expression.



We need to expand and combine like terms as usual :

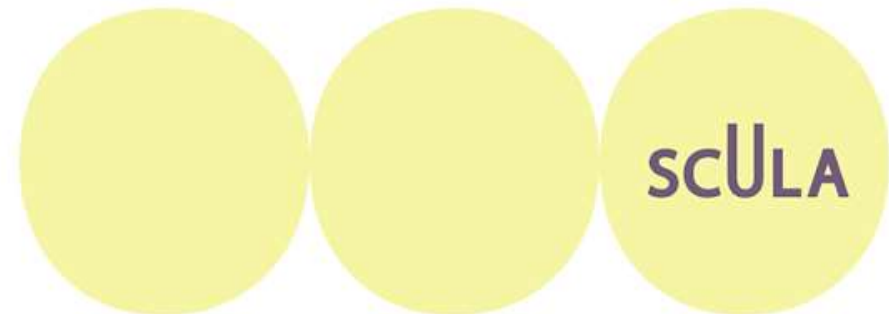
$$S = (4 + i)(5 - 2i)$$

$$S = 20 - 8i + 5i + 2$$

$$S = 22 - 3i$$

Therefore, the answer is B

Remember the SAT will only require you to manipulate the product of two complex numbers.



The Conjugate of a Complex Number :

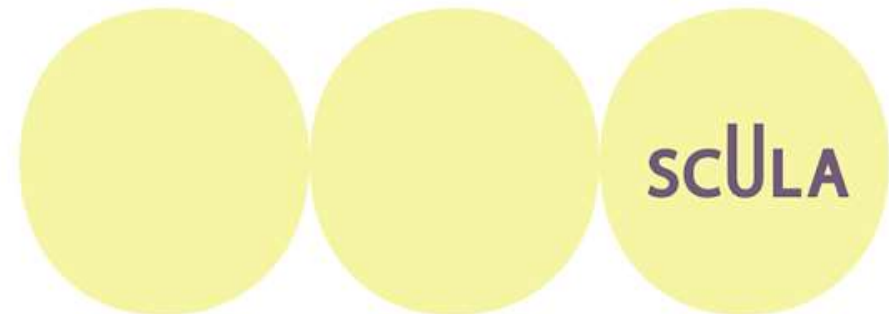
If $z = a + ib$ is a complex number, then the conjugate of this number is

$\bar{z} = a - ib$ and is noted \bar{z} (Z bar).

The sum of a number and its conjugate is a real number

$$z + \bar{z} = a + ib + a - ib$$

$$z + \bar{z} = 2a$$

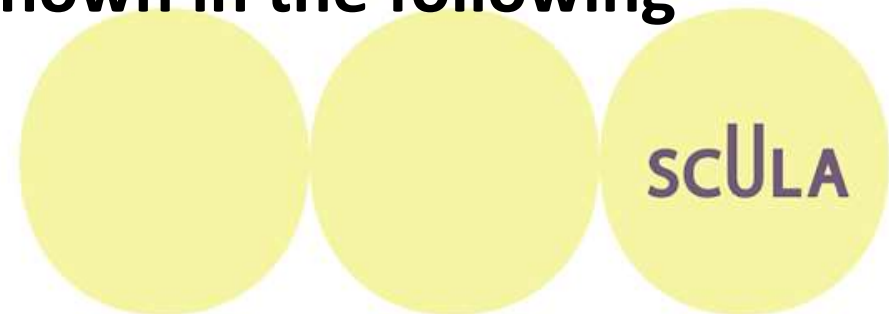


The product of a number and its conjugate is a real number

$$z \times \bar{z} = (a + ib)(a - ib)$$

$$z + \bar{z} = a^2 + b^2$$

Some questions will ask you to write the given numbers in the form of $a + ib$. For that, we will use the conjugate as shown in the following example



EXAMPLE 3: Which of the following is equal to $\frac{2+3i}{1+i}$?

A) $\frac{1}{2} - \frac{1}{2}i$

B) $\frac{1}{2} + \frac{1}{2}i$

C) $\frac{5}{2} - \frac{1}{2}i$

D) $\frac{5}{2} + \frac{1}{2}i$

To remove the complex number from the denominator, we will use the conjugate .

$$\frac{2 + 3i}{1 + i} = \frac{(2+3i)(1 - i)}{(1 + i)(1 - i)}$$

Then we will simplify both the denominator and the numerator .

$$\frac{2 + 3i}{1 + i} = \frac{2 + 3i - 2i + 3}{1 + 1}$$

Finally, we should write the fraction in the form of $a + ib$ where a and b are real numbers

$$\frac{2 + 3i}{1 + i} = \frac{5 + i}{2} = \frac{5}{2} + \frac{1}{2}i$$

PRACTICE

https://drive.google.com/file/d/1TUZJv9QkZFxBnBK5QjV_ckpsFRWONia_knil_evird=psu?weiv/0



THANK YOU!

DO YOU HAVE ANY QUESTIONS?

