SAT MATH SECTION

Complex Numbers

Today, we will discuss a new concept .

- Definition of Complex Numbers
- Sum of Complex Numbers
- Product of Complex Numbers
- Conjugate of a Complex Number

SCULA

REAL NUMBER

VS .

COMPLEX NUMBERS

We will start with a familiar situation.

Remember the quadratic functions that did not have any real solutions?

This is because they came down to an equation that looks like this:

$$x^2 + 1 = 0$$

$$x^2 = -1$$

This equation, indeed, does not have real solutions.

There is no real number squared that is equal to -1 or any

negative number. Squared numbers are positive or equal to zero.

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Introduction of the number *i*

To solve similar equations, Gerolamo Cardano invented an imaginary number, referred to a i for which this previous equation apply.

$$i^2 = -1$$

With the introduction of this number, we can find complex solutions that are made of the imaginary number i.

For example,

$$x^2 = -5 = -1 \times (\sqrt{5})^2$$

$$x^2 = -5 = i^2 \times (\sqrt{5})^2$$

Therefore ,

$x = i\sqrt{5}$ or $x = -i\sqrt{5}$

The previous equation has complex solutions but no real solutions .

$i\sqrt{5}$ is a complex number .

In general, any number written in the form of $z = a + ib$ where a and b are real numbers is called a complex number

From there, any power of i can be defined as follow :

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i$

$i^6 = -1$

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z = a + ib is a complex number.

a is called the real part (because a is a real number)

ib is the imaginary part (because b is multiplied by the imaginary number b).

Every real number is a complex number with an imaginary part equal to 0.

Complex numbers cannot, on the other hand, be real numbers.

The Sum of Complex Numbers:

The sum of two complex numbers is a complex number.

Let's consider z and z' two complex numbers.

$z = a + ib$

$z' = a' + ib'$

$z + z' = a + ib + a' + ib'$

$$\mathbf{z + z' = (a + a') + i(b + b')}$$

EXAMPLE 1: If $i = \sqrt{-1}$, which of the following is equivalent to $(3 + 5i) - (2 - 3i)$?

A) $9i$ B) $1 + 2i$ C) $1 + 8i$ D) $5 + 8i$

The sum and the subtraction of two complex numbers or more follows the same rules. We sum the real parts and imaginary parts to have a complex number in the form of a + ib

If a = 0 then the complex number will be in the form of ib .

If b = 0 then the complex number will be a real number a

We need to expand and combine like terms as usual :

$S = (3 + 5i) - (2 - 3i)$

$S = 3 + 5i - 2 + 3i$

$S = 1 + 8i$

Therefore, the answer is C

Remember the SAT will only require you to manipulate the sum or subtraction of complex numbers.

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The Product of Complex Numbers:

The product of two complex numbers is also a complex number.

Let's consider z and z' two complex numbers.

$z = a + ib$

$z' = a' + ib'$

$z \times z' = (a + ib)(a' + ib')$

$$\mathbf{z\,z' = aa' + iab' + ia'b - bb'}$$

$$\mathbf{z\,z' = (aa' - bb') + i(ab' + a'b)}$$

EXAMPLE 2: Given that $i = \sqrt{-1}$, what is the product $(4 + i)(5 - 2i)$?

A) $18 - 3i$ B) $22 - 3i$ C) $18 + 3i$ D) $22 + 3i$

We deal with the product, as we discussed, in the same way that we expand any product. Then we combine like terms to have a complex number expression.

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We need to expand and combine like terms as usual :

$S = (4 + i)(5 - 2i)$

$S = 20 - 8i + 5i + 2$

$S = 22 - 3i$

Therefore, the answer is B

Remember the SAT will only require you to manipulate the product of two complex numbers.

The Conjugate of a Complex Number :

If $z = a + ib$ is a complex number, then the conjugate of this number is

$\bar{z} = a - ib$ and is noted $\bar{z}$ (Z bar).

<u>The sum of a number and its conjugate is a real number</u>

$z + \bar{z} = a + ib + a - ib$

$z + \bar{z} = 2a$

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The product of a number and its conjugate is a real number

$z \times \overline{z} = (a + ib)(a - ib)$

$z + \overline{z} = a^2 + b^2$

Some questions will ask you to write the given numbers in the form of a + ib. For that, we will use the conjugate as shown in the following example

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EXAMPLE 3: Which of the following is equal to $\frac{2+3i}{1+i}$?

A) $\frac{1}{2}-\frac{1}{2}i$ B) $\frac{1}{2}+\frac{1}{2}i$ C) $\frac{5}{2}-\frac{1}{2}i$ D) $\frac{5}{2}+\frac{1}{2}i$

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To remove the complex number from the denominator, we will use the conjugate .

$$\frac{2+3i}{1+i} = \frac{(2+3i)(1-i)}{(1+i)(1-i)}$$

Then we will simplify both the denominator and the numerator .

$$\frac{2+3i}{1+i} = \frac{2+3i-2i+3}{1+1}$$

Finally, we should write the fraction in the form of a + ib where a and b are real numbers

$$\frac{2+3i}{1+i} = \frac{5+i}{2} = \frac{5}{2} + \frac{1}{2}i$$

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PRACTICE

https://drive.google.com/file/d/1TUZJv9QkZFxBnBK5QjV_ckpsFRWONia knil_evird=psu?weiv/0

THANK YOU!

DO YOU HAVE ANY QUESTIONS?